II B. TECH II SEMESTER SUPPLEMENTARY EXAMINATIONS JULY - 2022
RANDOM VARIABLES AND STOCHASTIC PROCESSES
(ELECTRONICS AND COMMUNICATION ENGINEERING)
Time: 3 hours
Max. Marks: 60

Note: Answer ONE question from each unit (5 $\times 12$ = $\mathbf{6 0}$ Marks)

## UNIT - I

1. a) A random variable $X$ has a density function $f_{X}(x)=C\left(1-x^{4}\right) \quad[6 M]$ in $-1 \leq \mathrm{x} \leq 1$. Find the value of ' C ' and $\mathrm{P}[|\mathrm{x}|<0.5]$.
b) State the properties of density and distribution functions.
2. a) If $X$ is Gaussian random variable, show that

$$
\int_{\infty}^{\infty} x f_{\mathrm{x}}(\mathrm{x})=\mathrm{m}_{\mathrm{x}}
$$

b) Write the properties of Conditional density and distribution function of a random variable.
UNIT - II
3. a) A random variable $X$ can have values $-4,-1,2,3$, and 4, each with probability 0.2 . Find (i) the density function (ii) the mean (iii) the variance of the random variable $\mathrm{Y}=\mathrm{X}^{2}$.
b) Find the expected value of the function $g(X)=X^{3}$ where X is a random variable defined by the density.

$$
\begin{equation*}
f_{X}(x)=\left(\frac{1}{2}\right) u(x) \exp (-x / 2) \tag{OR}
\end{equation*}
$$

4. a) Find the characteristic function and the first two moments for $f_{X}(x)=a e^{-b x}, x \geq 0$.
b) Explain the transformations of a random variable X .
UNIT - III
5. a) Define Marginal density function? Find the Marginal density functions of with joint density function.

$$
f_{X Y}=\frac{1}{12} u(x) u(y) e^{-x / 3} e^{-y / 4}
$$

b) Gaussian random variables $X$ and $Y$ have first and second order moments $\mathrm{m}_{10}=-1.1, \mathrm{~m}_{20}=1.16, \mathrm{~m}_{01}=1.5, \mathrm{~m}_{02}=2.89$, $\mathrm{R}_{\mathrm{XY}}=-1.724$ Find $\mathrm{C}_{\mathrm{XY}}, \rho$
6. a) Defined the random variables $V$ and $W$ by (i) $V=X+a Y$ (ii) $\mathrm{W}=\mathrm{X}$-aY Where ' $a$ ' is real number and X and Y random variables, Determine 'a' in terms of X and Y such that V and W are orthogonal?
b) Given the joint distribution function
$F_{X, Y}(x, y)=\left[1-e^{-a x}-e^{-a y}+e^{-a(x+y)}\right] u(x) \cdot u(y)$
Find the conditional density functions $\mathrm{f}_{\mathrm{X}}(\mathrm{x} / \mathrm{y})$ and $\mathrm{f}_{\mathrm{Y}}(\mathrm{y} / \mathrm{x})$. Are the random variables X and Y statistically independent.
UNIT -IV
7. a) Explain the classification of Random processes.
b) Explain the properties of cross correlation functions
8. a) Consider a random processes $X(t)=A \cos \left(\omega_{1} t+\theta\right)$ and
$\mathrm{Y}(\mathrm{t})=\mathrm{B} \cos \left(\omega_{2} \mathrm{t}+\phi\right)$ where $\mathrm{A}, \mathrm{B}, \omega_{1}$ and $\omega_{2}$ are constants while $\theta$ and $\phi$ are statistically independent random variables uniformly distributed on $(0,2 \pi)$. Show that $X(t)$ and $Y(t)$ are jointly WSS processes.
b) If $\theta=\phi$, show that the two processes are not jointly WSS unless $\omega_{1}=\omega_{2}$.
UNIT -V
9. a) A random noise $X(t)$, having a power spectrum
$\mathrm{S}_{\mathrm{XX}}(\omega)=9 /\left(49+\omega^{2}\right)$ is applied to a differentiator that has a transfer function $H_{1}(\mathrm{w})=\mathrm{jw}$, the differentiator's output is applied to a network for which $\mathrm{h}_{2}(\mathrm{t})=\mathrm{u}(\mathrm{t}) . \mathrm{t}^{2} \exp (-7 \mathrm{t})$ and the network's response is a noise denoted by $\mathrm{Y}(\mathrm{t})$. Find the average power in $\mathrm{X}(\mathrm{t})$
b) Find the power spectrum of $Y(t)$.
(OR)
10. a) Define the following terms. (i)Noise equivalent temperature (ii) Noise figure (iii) Available power gain.
b) The noise present at the input to a two port network is $1 \mu \mathrm{~W}$. the noise figure $F$ is 0.5 dB , the receiver gain $\mathrm{g}_{\mathrm{a}}=10^{10}$, calculate:
(i) The available noise power contributed by two port network
(ii) The output available power.

